Modulation transfer function of a lens measured with a random target method

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We measured the modulation transfer function (MTF) of a lens in the visible region using a random test target generated on a computer screen. This is a simple method to determine the entire MTF curve in one measurement. The lens was obscured by several masks so that the measurements could be compared with the theoretically calculated MTF. Excellent agreement was obtained. Measurement noise was reduced by use of a large number of targets generated on the screen. © 1999 Optical Society of America

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1. Introduction

The modulation transfer function (MTF) of an optical system is an accepted way of describing its optical properties.[1] Its convenience results from the simple conversion of the convolution operation in the image space to multiplication in the spatial frequency domain. Describing a system by its MTF is most useful when the optics are complicated, in which case it gives a good measure of the spatial resolution attainable, and deviations of the measured MTF from a desired MTF can easily be quantified.

In this paper we present measurements of the MTF of a lens in the visible region by a recently proposed technique using a random target. The experimental measurements are compared with the MTF as calculated for the same lens, and excellent agreement is demonstrated.

Many MTF measurement systems involving sampled images encounter a major problem: they are not shift invariant.[2] These measurements are affected by the position of the test target (usually a point source or a line source) with respect to the rows and columns of the detector array. An alternative shift-invariant method is the scanning-knife-edge technique,[3] which involves scanning focal plane arrays. The random transparency target technique has been demonstrated as a shift-invariant method of measuring MTF without requiring mechanical scanning.[4,5]

The random-target method, as described by Daniels *et al.*,[4] is able to evaluate the entire MTF curve in a single measurement with only one target. It uses a random test target produced by a printer on a transparency, illuminated by a broadband incoherent source and then imaged by a lens onto a CCD array. After data analysis the MTF curve is presented.

The experiments presented here benefit from all the advantages of the random-target method: shift invariance, testing of the entire spatial frequency range in one measurement, and use of truly incoherent light. In particular, we show that the MTF of the lens alone can also be measured by this method, which was not done previously. Our method also differs from previous research in the following manner. The incoherent light source used is an ordinary computer screen showing the random pattern placed in the object position. A major advantage of this method is the simplicity of target preparation. First, there is no need to take into account an additional MTF of the printing process. Second, a large number of targets can easily be created and their results averaged to reduce noise. Third, it is easy to filter (by computer) the target spatial frequencies and measure only within the band of interest. Fourth, the size of the target can easily be changed to match the camera. In this study we concentrated on measuring the MTF of the imaging lens system itself, which can be evaluated theoretically, independently of the properties of the CCD camera being used. The

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horizontal line structure of the screen image can be ignored if measurements are restricted to the x coordinate.

2. Experimental Method

A. Hardware

The 14-in. computer screen was situated at a distance of 5 m from the imaging lens; the optical axis was folded with a plane mirror. The CCD situated in the image plane (Sony Model SPFM102CE) had 500×582 pixels, but the object size was adjusted so that the pixel array illuminated had a commensurate dimension (400×400) to the object array (200×200). The lens used for these experiments was a simple achromat with an effective focal length of 160 mm. For demonstrating results that depended on the lens itself, the lens was masked in various ways so that the theoretical MTF could be calculated and compared with the experimental results. This enabled us to show that significant quantitative results could be obtained. Images were captured with a IMAGE-PRO PLUS (Media Cybernetics) frame grabber and software. The calculations were done with MATLAB.

B. Design of Random Targets

The measurements require a random two-dimensional radiance pattern whose spectral distribution is that of white noise. A common computer random number generator is used to generate an array of $N \times N$ random numbers. When this $N \times N$ matrix is imaged by the computer in gray scale it becomes a two-dimensional random pattern with a band-limited spatial frequency white-noise distribution. The bandlimit is

$$f_{\max} = 1/2l = 28.4 \text{ cycles/mm}, \tag{1}$$

where l is the pixel spacing of the target as measured in the image plane:

$$l = h_{\text{obj}} M/N = 0.0176 \text{ mm}, \tag{2}$$

where h_{obj} is the length of the random target, N is the number of pixels in each row of the target, and M is the magnification of the optical system. In our system $M = 0.0151$. One such random target is shown in Fig. 1(a).

C. Data Analysis

One easily calculates the power spectrum density (PSD) by performing the two-dimensional $|\text{FFT}|^2$ operation on the matrix and averaging over the rows. Figure 1(b) shows that the target has the desired PSD in a one-dimensional projection (denoted by PSD_i).

For the MTF of the complete system to be obtained it is necessary to relate PSD_i to the final PSD_f measured by the CCD. Because the target consists of a (200×200) matrix, the image detected by the CCD (400×400) must first be converted to a matrix of the same dimension. The final and the initial PSD's at spatial frequency f obey the equation

$$\text{PSD}_f(f) = \text{MTF}_{\text{tot}}{}^2(f)\text{PSD}_i(f), \tag{3}$$

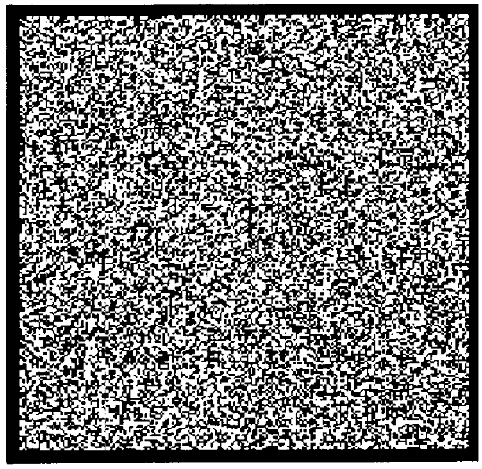
where MTF_{tot} is the MTF of all the optics from the computer screen through the CCD array. This means that the ratio $\text{PSD}_f/\text{PSD}_i$ results from only the system MTF.

For incoherent illumination the measured MTF_{tot} is the product of the MTF_{sys} of the optical setup (screen and CCD) and the MTF_{test} of the tested optical element. Furthermore, when the initial PSD consists of only the zero frequency (a homogenous gray target) there is an output $\text{PSD}_f(f)$ caused by the noise amplitude spectrum (NAS) typical of the system, so that we can write

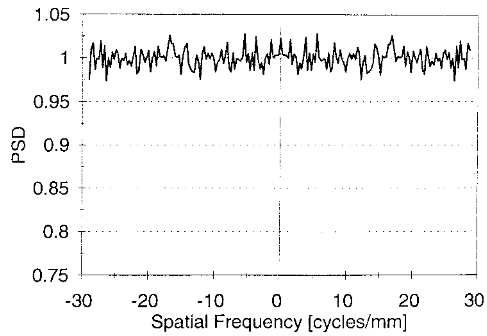
$$\text{PSD}_f(f) = [\text{MTF}_{\text{sys}}(f)\text{MTF}_{\text{test}}(f)]^2\text{PSD}_i(f) + \text{NAS}(f),$$

$$\text{MTF}_{\text{test}}{}^2(f) = \frac{[\text{PSD}_f(f) - \text{NAS}(f)]}{\text{MTF}_{\text{sys}}{}^2(f)\text{PSD}_i(f)}. \tag{4}$$

Note that division by $\text{PSD}_i(f)$ is only a means of improving the signal-to-noise ratio and does not change the MTF curve, because $\text{PSD}_i(f) \sim 1$ for all frequencies.



(a)



(b)

Fig. 1. (a) Band-limited white-noise random target. (b) PSD_i of a random target.

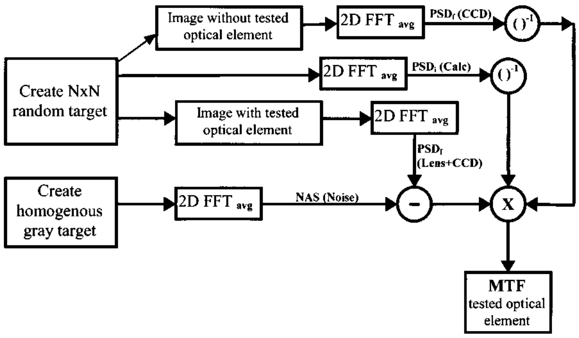


Fig. 2. Block diagram for the experimental method.

D. Methods of Measurement

In this study two types of MTF measurements were made:

(i) Continuous MTF measurement, which gives the whole MTF curve within the band limit of the random target. For this measurement it is necessary to measure the NAS in a separate experiment.

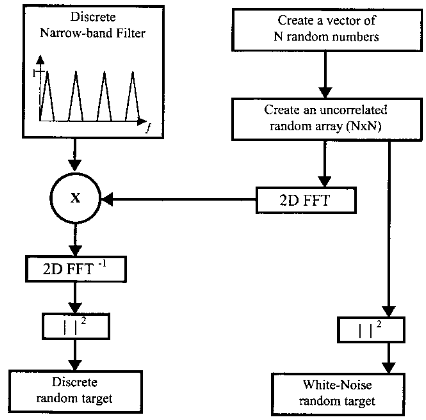
(ii) Discrete MTF measurement, which samples the MTF curve at certain discrete frequencies. The advantage of the discrete random-target method is the ability to sample both the MTF curve and the NAS in one measurement, since the noise amplitude is obtained at frequencies between those passed by the filter.

The disadvantage of the discrete measurements is that the MTF is measured only at certain discrete spatial frequencies, and the complete curve has to be obtained by interpolation. This may be inadequate for certain applications.

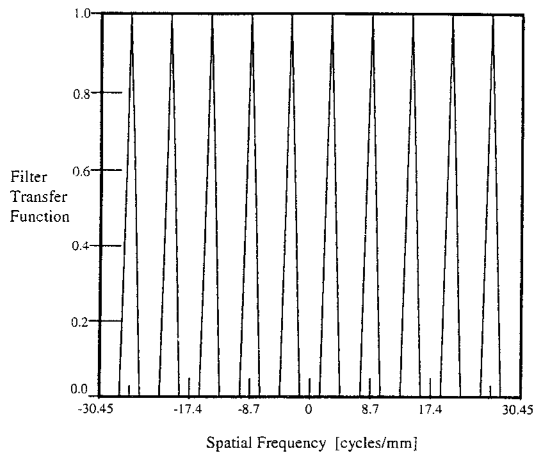
In both types of measurement the experimental process is the same, as shown in the block diagram in Fig. 2. If the random target is left unfiltered the result is a continuous MTF curve, and if the target is filtered so that it would feature only discrete spatial frequencies, the result is the discrete MTF measurement. Block diagrams describing the filtering process and a plot of the filter used are shown in Figs. 3(a) and 3(b), respectively. Figure 4 shows an image of the calculated discrete random target. The measurements were performed with a set of 15 unfiltered random targets for the continuous measurement and another set of 15 filtered targets for discrete MTF measurements. The results were calculated for each target individually and then averaged.

3. Results

As we described, we chose to demonstrate the capabilities of this measurement system by measuring the MTF of a set of apertures masking the imaging lens. To measure the NAS of our system, we measured the PSD_f originating from the PSD_i of a homogenous gray target. To find the MTF of the measurement system (MTF_{sys}), we performed a measurement using the random targets with the full aperture of the lens, in which case the MTF_{sys} is limited by the camera.[3] These two results are presented in Figs. 5(a) and 5(b), respectively.



(a)



(b)

Fig. 3. (a) Block diagram for the generation of random targets. (b) Discrete filter transfer function. This filter is the convolution (*) of comb $(f/5.8 - 1/2) * \mathrm{trg}(f/1.45)$, where $\mathrm{trg}(x) = 1 - |x|$ or zero if $1 - |x| < 0$.

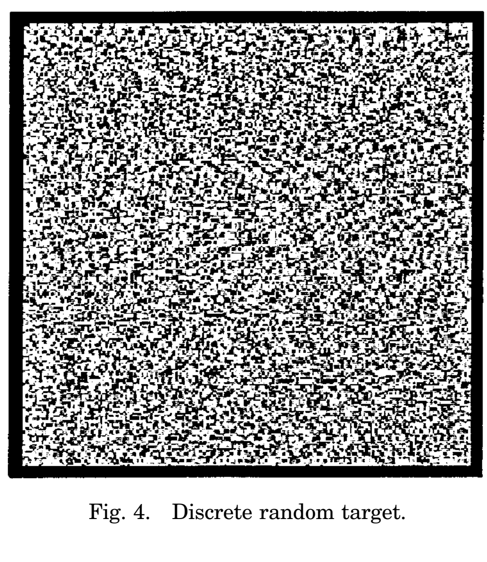
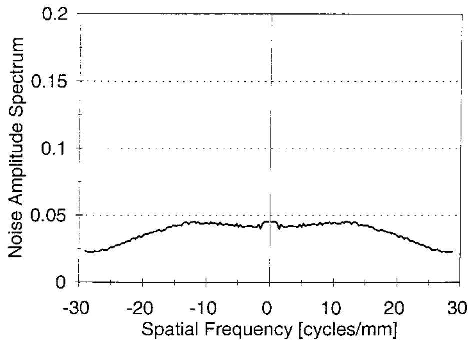
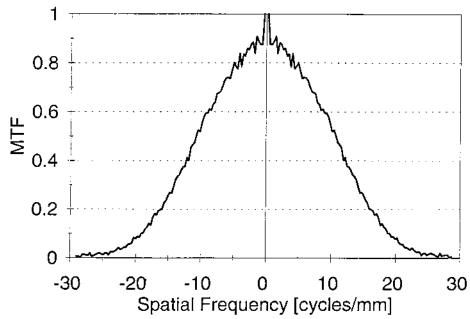


Fig. 4. Discrete random target.

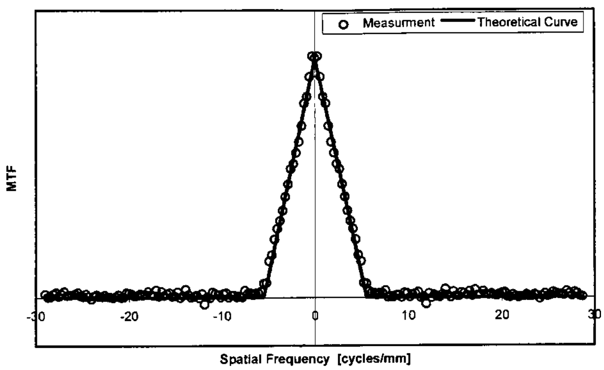


(a)

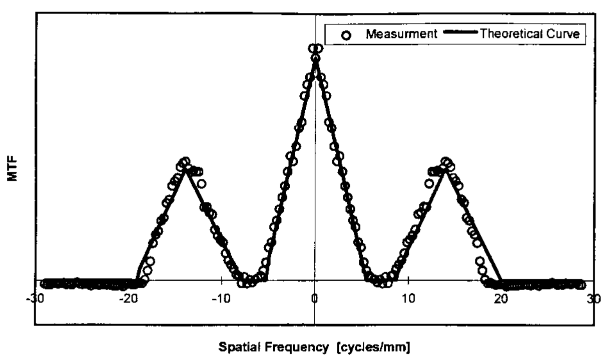


(b)

Fig. 5. (a) NAS obtained by a homogenous gray target. (b) MTF_{sys}, the measurement of which was carried out with the full aperture of the lens.



(a)



(b)

Fig. 6. Continuous MTF of (a) a single-slit aperture and (b) a double-slit aperture: experimental results (circles) and theoretical curve (solid curve).

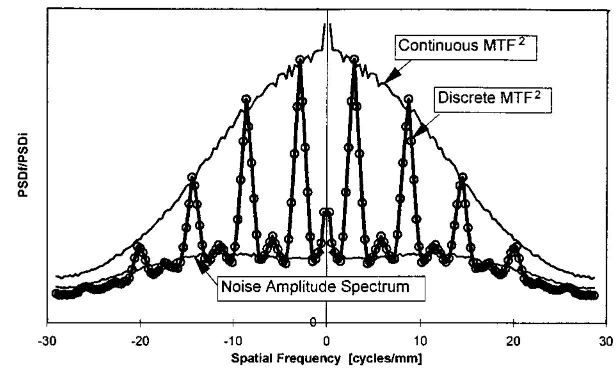
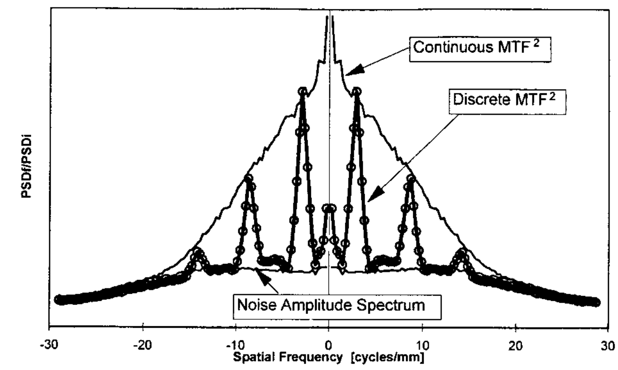
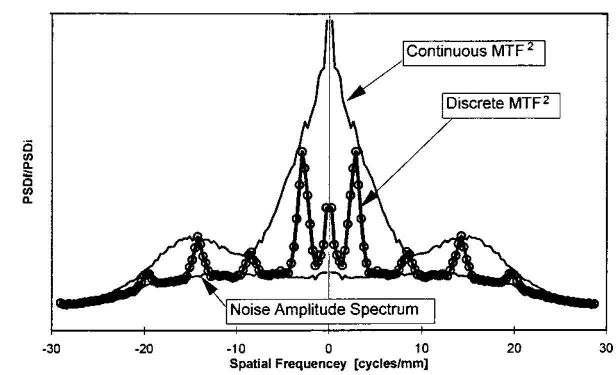


Fig. 7. Discrete PSD measurements (circles) compared with the continuous measurements (upper curve) shown in Fig. 5(b) and the NAS (lower curve) in Fig. 5(a). The discrete measurements sample the continuous MTF at the peaks.



(a)



(b)

Fig. 8. Same as Fig. 7(a) for a single-slit aperture and (b) a double-slit aperture.

A. Continuous Modulation Transfer Function Measurement

It was difficult to obtain reliable results on the lens alone when it was masked by a circular aperture. If the diameter of the aperture was sufficiently reduced to make the cutoff frequency of the lens MTF less than $1/2l$, the intensity of the screen was insufficient to make reliable measurements. For larger apertures what is measured is determined by the CCD and not the lens (compare Ref. 3). We therefore chose to use slit-type masks for which the lens MTF could be calculated; the narrow width of the slit resulted in significant measurements, while the larger height of the slit allowed sufficient light to be transmitted. Two examples were examined. The first was a single-vertical-slit aperture symmetrically disposed about the optical axis. A slit aperture gives a triangular MTF curve[1]:

$$\mathrm{MTF} = \begin{cases} 1 - |f/f_a| & \text{for} \quad |f| \le f_a \\ 0 & \text{for} \quad |f| > f_a \end{cases}; f_a = \frac{d}{F\lambda}, \tag{5}$$

where d is the width of the slit, λ is the mean wavelength emitted by the computer screen (0.55 μm), and F is the focal length of the lens.

The second example was a pair of slits, also symmetrically disposed, that gives three triangular peaks, the center one being twice the height of the others:

$$\mathrm{MTF} = \begin{cases} 1 - |f/f_a| & \text{for} & |f| \le f_a \\ |f/2f_a| - (f_b + f_a)/2f_a & \text{for} & f_a + f_b \le |f| \le 2f_a + f_b \\ (3f_a + f_b)/2f_a - |f/2f_a| & \text{for} & 2f_a + f_b \le |f| \le 3f_a + f_b \\ 0 & \text{for} & \text{else} \end{cases}; f_b = \frac{d_1 - d}{F\lambda}, \tag{6}$$

where d_1 is the distance between the two slits.

Two sets of slits were used: (1) the single-slit configuration was 0.5 mm in width and 35 mm in height, and the double-slit configuration consisted of two such slits separated by 0.7 mm horizontally; (2) the single-slit configuration was 1.5 mm in width and 35 mm in height, and the double-slit configuration consisted of two slits 0.75 mm in width and 35 mm in height separated by 0.7 mm horizontally.

The experimental results are shown in Figs. 6(a) and 6(b) for set of slits (1) compared with the calculated values. There is an excellent match between the theoretical curve and the experimental results. It is also noticeable that the widths of the central triangular peak, both in the single- and double-slit apertures, are identical. This follows from the fact that the dimensions of the slits are also identical. Because the zero-frequency response cannot be obtained experimentally, it was obtained by extrapolation of the low-frequency response, and the results were normalized to unity at this point. There are no adjustable parameters in the comparison.

B. Discrete Modulation Transfer Function Measurement

Using the discrete random targets we performed measurements without any aperture and with single- and double-slit apertures.

The discrete measurement of $\mathrm{MTF}_{\mathrm{sys}}$ is shown in Fig. 7. The results of the continuous and the discrete methods and the spatial NAS are superimposed in this figure. Note how the discrete measurement curve samples the continuous measurement curve at discrete spatial frequencies, and how the NAS matches the levels in the guardbands between the triangles. This kind of measurement shows two characteristics of the system in one plot.

Figures 8(a) and 8(b) show the discrete and continuous measurement results for set of slits (2) superimposed, along with the spatial NAS. There is a very good correlation between the three curves. Slit pattern (2) was used for this comparison because it gave a larger number of measurement points.

4. Discussion and Conclusions

We have clearly shown that this method of measuring MTF is reliable and shift invariant and allows determination of the entire MTF curve with one measurement. The simple optical setup allows measurement both of the system MTF and of the combined MTF of the tested element and the system. This allows isolation of the desired MTF of the optical element.

Using a computer screen imaging a random matrix as a radiance pattern simplifies measurement and analysis. However, there is a need to increase the screen intensity to perform measurements on optical elements with small apertures. The method is also limited, in practice, by the maximum spatial frequency of the CCD, which is usually considerably smaller than the lens capability.

In principle, the complex optical transfer function could be measured by the same technique. Instead of the PSD, the complex amplitude spectra would be measured, and the relationship equivalent to Eq. (1) is

$$\mathrm{ASF}_f(f) = \mathrm{OTF}[\mathrm{ASF}_i(f)].$$

This was not attempted in the present study.

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