

# Low modal volume dipole-like dielectric slab resonator

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**Abstract:** We propose a novel geometry in a silicon planar resonator with an ultra-small modal volume of  $0.01(\lambda/2n)^3$ . The geometry induces strong electric field discontinuities to decrease the modal volume of the cavity below  $1(\lambda/2n)^3$ . The proposed structure and other common resonators such as 1D and 2D photonic crystal resonators are compared for tradeoffs in confinement and quality factors.

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**OCIS codes:** (130.0130) Integrated optics; (230.5750) Resonators; (130.3120) Integrated optical devices

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## 1. Introduction

Electromagnetic resonant cavities are a critical element for optical devices, such as lasers, filters and switches. The cavities are characterized by modal volume,  $V$  [1, 2] and quality

factor,  $Q$ . High  $Q/V$  ratio is important in increasing light-matter interactions in processes such as spontaneous emission, nonlinear optical processes and strong coupling. The control of these interactions is essential in low-threshold nanolasers [3], quantum information processing devices [4] and photonic chips. Small volume plasmonic cavities have been suggested [5, 6], however the presence of the metal increases fundamental absorption and limits the  $Q$  values. Dielectric structures with high  $Q$  and small  $V$  have been demonstrated, most with diffraction limited modal volumes [7-11] or low  $Q$ s [12]. Devices which operate on fast time scales and high bandwidths can be penalized by high  $Q$  cavities, which have long loading times and narrow operating bandwidths. Narrow bandwidth cavities tend to be more sensitive to temperature variations which increase fluctuations of resonant frequencies. Here we propose a resonator with single dielectric layer geometry with an ultra-small modal volume to decrease dependence on high  $Q$  for  $Q/V$  critical devices.

The challenge of low volume of dielectric cavities lies in the fact that as the modal volume of the light in the cavity approached the diffraction limit, the diffraction losses near the reflector region increase and consequently decreases the  $Q$  and decentralizes the optical mode. In a homogenous dielectric medium a propagating plane wave is diffraction limited in its width to  $\sim\lambda/2n$ , where  $n$  is the refractive index of the dielectric. Micro photonic devices are typically limited to dielectric materials with indices up to  $n=3.45$  (Si). A cavity with two perfect mirrors spaced half wavelength apart thus has a resonant mode volume of no less than  $1(\lambda/2n)^3$ . Low modal volume holey waveguide (HW) cavities have been shown before [12], consisting of 2 dielectric Bragg mirrors inside of a Si waveguide, the modal volume was achieved as low as  $0.042(\lambda/2n)^3$  when a low index perturbation was introduced in the cavity.

## 2. Theory

The cavity proposed here with modal volume of  $0.01(\lambda/2n)^3$  and  $10^3$   $Q$  was inspired by a structure found using an evolutionary algorithm (EA). An EA is a stochastic search employed to probe large multidimensional spaces to find global maxima. The algorithm constructs a design solution (shape) by repeatedly selecting, varying, and replicating successful individuals from a population of candidate solutions. The initial solution candidates are completely random structures, Fig. 1(a). During each cycle of the algorithm, possible solutions are evaluated and assigned a merit value. The evaluated solutions are ordered by merit value and the poorest performing solutions are discarded, a new set of solutions are formed by mixing various attributes of evaluated solutions and then further randomly modified. The algorithm then starts a new iteration with the new solutions, Fig. 1(b-d). Figure 1 is a schematic of a planar dielectric cavity designed with an evolutionary algorithm (EA).

We represent the shape of a resonator as a  $120 \times 120$  matrix of binary values, corresponding to high and low index dielectric in the slab. The merit function for a low modal volume cavity is the value of the field amplitude in the center of the optical mode in a steady state with continuous wave excitation. The cavity is excited via normally coupled waveguide (top of Fig. 1(a-d)). The excitation is simulated using a finite difference time domain (FDTD) algorithm in 3D. The EA ran on a 2000 CPU Opteron based super computer for 12 hours to arrive at Fig. 1(d).

Maximizing steady state field at a single point should favor resonant modes akin to a delta function. This approach may not necessarily produce the highest  $Q/V$  cavity, but should give us insight into ultra low modal volume cavities. Our previous work [15] with a 2D EA with the same merit function produced comparable results showing reflection symmetry along  $x$  and  $y$  axes. We employed these symmetries in this algorithm to reduce the iteration time. We found that discretizing the device spacially with a 40nm grid size was sufficient for convergence. A finer resolution did not lead to a qualitatively different device and exponentially lengthened algorithm execution time. A coarser resolution introduced excessive scattering and did lead to a highly localized field.

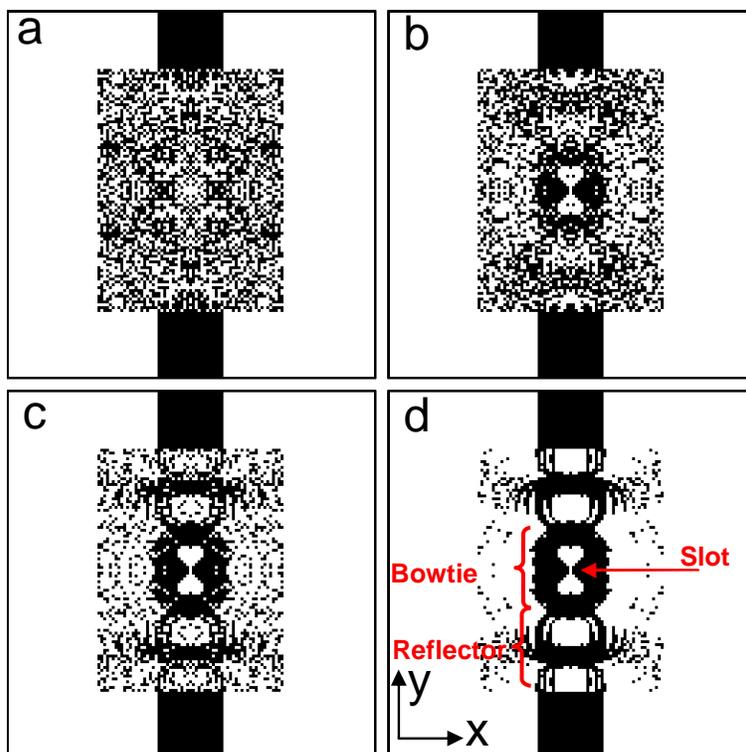


Fig. 1. Evolution of a planar resonator in an evolutionary algorithm; (a) 1<sup>st</sup> generation, completely random device; (b) 100<sup>th</sup> generation, the bowtie shape is defined, (c) 200<sup>th</sup> generation, the bowtie shape is well defined and grating like structure begins to emerge; (d) 800<sup>th</sup> generation, the bowtie and the grating like structure are cleanly defined; each image is 120x120 pixels, each pixel represents 40x40nm square peg 250nm high in a 3D finite difference time domain simulation.

The evolutionary algorithm can employ a variety of approaches to mix and alter candidate solutions. We found that mixing randomly selected rectangular subsections of the candidates was most effective for convergence. We also used a population of 100 solutions, where the best 30% of the candidates were kept in each iteration and the others were discarded. Our previous [15] work offers more details about the algorithm implementation.

The evolved cavity, Fig. 1(d), resembles a bowtie, Fig. 2(b), embedded inside of a 1-D photonic crystal waveguide. The high index material (3.45) is silicon and the low index material (1.45) is silicon dioxide. The cavity achieves an ultra-low modal volume with slot region shortened to a point in the middle of the bow-tie. In contrast to typical disk, ring, and photonic crystal cavities, the electric field is localized in the low index material. Photonic crystal cavities with low index defects have been demonstrated. Loncar et al. [16] demonstrated a cavity with a small hole defect, and Kwon et al. [17], recently proposed a slot with 3 holes defect photonic crystal cavity. Both of those approaches show strong field confinement in the low index region, but exact modal volume is not reported.

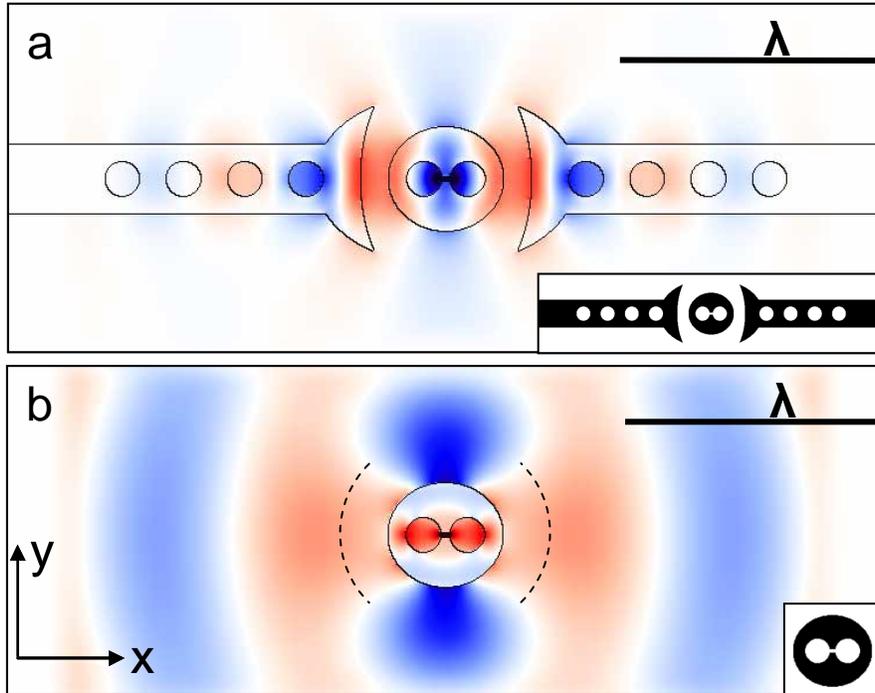


Fig. 2. (a) Resonant mode amplitude ( $E_y$ ) in a planar bowtie cavity; (b) radiative mode amplitude ( $E$ ) in a bowtie, the dotted line represents the curvature of radiating field; insets, dielectric geometry of the respective devices, white: low index oxide 1.45, black: high index silicon 3.45.

The geometry found utilizes electric field discontinuities to decrease the modal volume of the cavity below  $V_{eff} = \int \epsilon(r) |E(r)|^2 dr^3 / \epsilon(r_{max}) \max(|E(r)|^2) = 1(\lambda/2n)^3$ . Note that the  $n$  in units of modal volume  $(\lambda/2n)^3$  is the material index where the peak electric field is located. Displacement field normal to a dielectric interface must be continuous to satisfy the boundary conditions; consequently the electric field is discontinuous normal to an interface. The discontinuity leads to a field enhancement in the low index dielectric[13, 14]. This effect can be utilized in a narrow low index dielectric slot placed in a high index dielectric waveguide. The modal cross section area of a single mode silicon waveguide with a 40nm silicon oxide slot is  $\sim 0.02(\lambda/2)^2$ , significantly lower than the diffraction limit. A slot waveguide resonant cavity with Bragg mirrors have been measured to have  $Q$  of 300 and a calculated modal volume  $\sim 0.04(\lambda/2n)$ [12]. The bowtie center found with EA in Fig. 1(a), supports a radiating mode which utilized the slot effect to create a strong electric field ( $E_x$ ) enhancement in the center of the cavity.

The proposed a cavity inspired by the evolved structure, Fig. 1(d), is shown in the inset of Fig. 2(a). The center of the cavity is a bowtie shape to provide slot effect localization. The outer ellipsoid of the bowtie is shaped to minimize emission into the  $y$  direction. Figure 2b shows the field distribution in the bowtie only. One can see that the bowtie emits a dipole-like radiation and is not a strongly resonant cavity where a reasonable  $Q$  can not be realized. We address these losses by embedding the bowtie in a cavity with curvature of innermost surfaces matching the centermost nodes of the field radiating from the bowtie shown in Fig. 2(b) (dotted line). The radiative mode from bow-tie is therefore contained by the surrounding Bragg mirrors with a wide bandgap (100nm) matching the radiative wavelength. The coupling to the waveguides and the  $Q$  of the cavity can be controlled by the number of holes in the waveguides. The field intensity of the resonant mode is plotted longitudinally ( $x$ ) and transversely ( $y$ ) to the center of the cavity in Fig. 3. We note that in both dimensions the field

peak is confined below the half wavelength of light and approach a delta function. In the transverse direction the enhancement is provided via the slot effect discussed above. In the longitudinal direction the finite length of the slot limits localization length. The total field amplitude is mostly (>99%) composed of the  $E_y$  component and is coupled to the TE like propagating mode in the waveguide. The  $E_x$  and  $E_z$  field components do not make a significant contributions to total field amplitude and are not plotted.

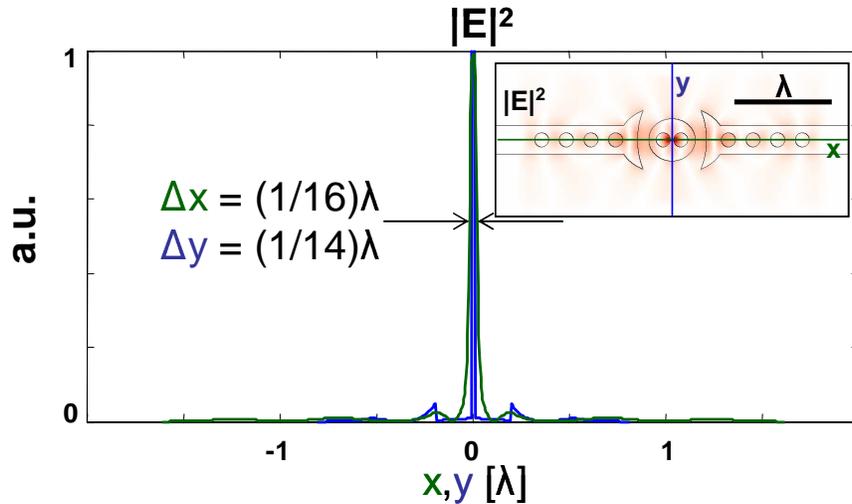


Fig. 3. Mode intensity in the bowtie cavity: blue, along the length and green in the transverse direction. The inset shows intensity distribution of a resonant mode in a bowtie cavity.

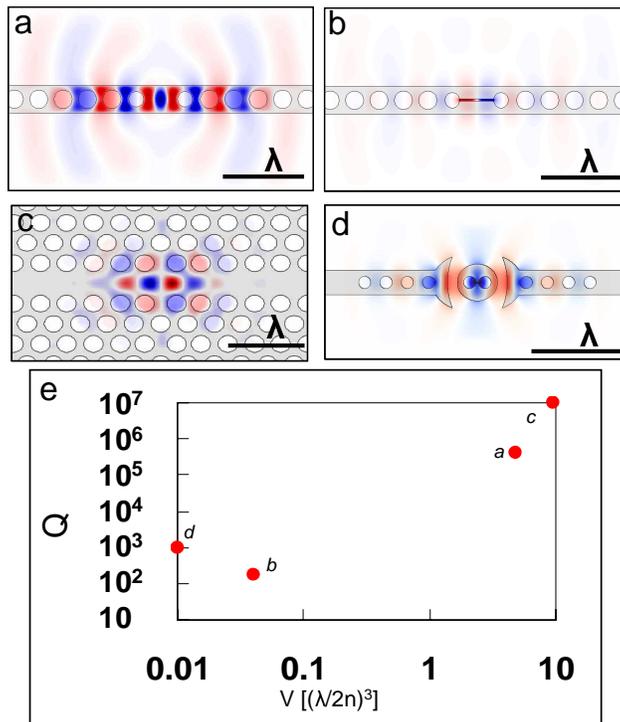


Fig. 4. (a-d), Resonant modes in various categories of planar cavities; (e)  $Q$  vs  $V$  values of the above cavities; (a) holey waveguide; (b) holey waveguide with slot; (c) photonic crystal heterostructure; (d) bowtie cavity;

### 3. Conclusion

High confinement planar cavities have been proposed in various configurations. The most common categories are photonic crystals with defects Fig. 4(a-c). Planar 2D photonic crystal cavities [7, 10, 11] have been proposed with modal volumes just above the diffraction limit ( $<10(\lambda/2n)^3$ ) with Qs as high as  $10^7$ . Holey waveguide cavities [9] have comparable modal volumes and Qs to the 2D photonic crystal cavities. Traveling wave cavities like disk, rings and spheroids provide ultra high Q but have relatively large ( $>10(\lambda/2n)^3$ ) modal volumes. The proposed planar bowtie cavity has a modal volume of  $0.01(\lambda/2n)^3$  and a Q of 1000. While other geometries can provide higher Q/V ratios, in applications where ultra high Q is undesirable due to long photon lifetimes or narrow resonance widths, strong light-matter interactions must depend on low modal volumes. Our design not only achieves a record small modal volume in a dielectric cavity but also confines majority of the field to the low index material, which can increase light-matter interaction with gasses and liquids.

### Acknowledgments

This work is partially supported by the Nanoscale Science and Engineering Initiative of the National Science Foundation under NSF Award # EEC-0646547 and the New York State Office of Science, Technology & Academic Research under NYSTAR Contract # C020071. This work is also partially supported by National Science Foundation (NSF) under grant Grant No. 0601460.