

Breakthroughs in Nonlinear Silicon Photonics 2011

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Abstract: We present an overview of the recent developments in nonlinear silicon photonics. There were significant advances in devices exploiting the $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearities. Here, we discuss parametric amplification, wavelength conversion, supercontinuum generation, parametric oscillation, and frequency comb generation in silicon-based devices.

Index Terms: Nonlinear optics, nanophotonics, silicon photonics, four-wave mixing (FWM).

1. Introduction

With the recent advances of fabrication and processing techniques, there has been significant progress in the development of silicon-based chip-scale devices for nonlinear optics. Efficient nonlinear optical processes require both large nonlinearities and low losses. The inherent large nonlinearity of silicon along with the high optical confinement that can be achieved due to the high refractive index contrast between the core and cladding allows for a significant enhancement in the effective nonlinearity, as compared with optical fibers [1]. This large effective nonlinearity combined with the improvement in linear propagation losses has allowed for extraordinarily compact nonlinear optical devices that previously might have required hundreds of meters of fiber.

In addition to the effective nonlinearity, the phase matching of the wavevectors associated with the interacting fields determines the efficiency of the nonlinear process. In order to achieve this phase matching, it is essential to engineer the effective dispersion of the nonlinear interaction. This dispersion depends on both the intrinsic material (material dispersion), which is fixed, and on the geometric properties of the medium (waveguide dispersion), which can be varied. In waveguide structures, the waveguide dispersion can be modified by engineering the cross section size and shape, allowing for the flexibility to tailor the net dispersion to be normal or anomalous to suit the particular nonlinear process [2]–[4].

In this review, we will focus on developments in 2011 related to silicon-based nonlinear optical devices utilizing the $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearity. These devices have shown promise largely due to the fact that they are CMOS-process compatible, are low-cost, have low linear losses in the telecommunication window, and offer promise for fully integrated optoelectronic devices. In particular, we will discuss the advances in parametric amplification and wavelength conversion, supercontinuum generation, and parametric oscillation and frequency comb generation.

2. Parametric Amplification and Wavelength Conversion

In recent years, there has been significant research in dispersion-engineered silicon waveguides for broadband parametric amplification and wavelength conversion. Parametric four-wave mixing (FWM) is a nonlinear process that depends on the third-order nonlinearity $\chi^{(3)}$ in which two pump photons annihilate to create a signal and idler photon pair. To achieve efficient FWM, it is necessary to tailor the dispersion of the waveguide such that the pump wavelength is tuned near the zero-group velocity dispersion (GVD) point of the waveguide [5], [6]. CMOS-compatible silicon-based FWM devices have been demonstrated using silicon, silicon nitride, and high-index silica glass waveguides and, over the past few years, there have been numerous signal processing applications utilizing chip-scale FWM, including signal regeneration [7], an ultrafast oscilloscope [8], and a radio-frequency spectrum analyzer [9]. Further developments in 2011 include real-time dispersion monitoring for high-speed differential phase-shift keying signals [10], spectral phase interferometry for direct electric-field reconstruction (SPIDER) [11], and time integration of optical signals [12]. For further development of these FWM-based devices, higher efficiencies and larger operating bandwidths are critical. Previous FWM demonstrations have shown extremely broadband wavelength conversion spanning from 1241 to 2078 nm using a pump laser in the telecommunications C-band [13]. The efficiency of the FWM process pumped in the C-band is ultimately limited by two-photon absorption (TPA) and free-carrier absorption (FCA) induced by TPA. Various techniques have been utilized to circumvent the effects of FCA, such as the use of a reverse-biased $p-i-n$ diode for carrier removal [14]. However, the effects of TPA are unavoidable at telecommunication wavelengths, due to the proximity to the silicon band-edge [15], [16].

Another silicon-based material that has shown promise for large parametric amplification is hydrogenated amorphous silicon (a-Si:H) [17]. Unlike crystalline silicon, it is possible to change the material properties of a-Si:H with fabrication procedures. In 2011, Kuyken *et al.* [18] demonstrated 26.5 dB parametric gain in a-Si:H waveguides. This is made possible since the bandgap of a-Si:H is 1.61 eV, in contrast with 1.12 eV for crystalline Si, which corresponds to a half-bandgap wavelength near 1550 nm. For a-Si:H this significantly reduces the effects of nonlinear absorption, allowing for an improvement in the figure-of-merit, defined as the ratio of the real and imaginary parts of the nonlinear parameter, by a factor of 4, as compared with crystalline Si waveguides [19]. While material degradation from pump exposure currently limits the performance [19], the a-Si:H platform offers potential for efficient on-chip optical signal processing.

Alternatively, there has been significant effort to design crystalline Si waveguides for operation near and beyond 2.2 μm , which corresponds to half the bandgap energy of silicon. Previously, large amplification was demonstrated with a pulsed pump source [20], [21]. In 2011, Lau *et al.* [22] presented the first demonstration of broadband continuous wave parametric wavelength conversion pumped near 2 μm . The results indicate a maximum conversion bandwidth of 748 nm, which is limited only by the detection range of the spectrum analyzer used. Simulation results indicate a theoretical bandwidth of 936 nm. Additionally, Kuyken *et al.* [23] demonstrated 50-dB parametric gain using a pulsed pump at 2173 nm. They achieve >40 dB gain over a wavelength range exceeding 580 nm. Recent studies performed in 2011 by Hon *et al.* [24] and Gholami *et al.* [25] indicate that the third-order nonlinearity in silicon beyond 2.2 μm is comparable to that in the near-infrared region. With the further development of platforms that exhibit lower losses in the midinfrared region, including silicon-on-porous-silicon and silicon-on-sapphire [26]–[28], the operation wavelength range of parametric mixing devices should extend into the 3–5- μm midinfrared region. Silicon photonics technology extended into this regime will allow for the translation of well-developed telecommunications technology to the midinfrared and the development of innovative devices for applications including astronomy, biochemical sensing, free-space communications, and spectroscopy.

In addition, there have been studies of parametric nonlinear processes based on the $\chi^{(2)}$ nonlinearity. Since in centrosymmetric materials, such as silicon, the bulk $\chi^{(2)}$ contribution vanishes, it is necessary to break the bulk symmetry to observe a second-order nonlinear response. Two approaches have been demonstrated. One utilizes a technique to induce strain in a silicon waveguide. To achieve the strain, the silicon waveguide is clad with a straining Si_3N_4 layer, allowing

for an efficient phase matching for $\chi^{(2)}$ interactions. Cazzanelli *et al.* [29] demonstrate both sum-frequency generation and second harmonic generation (SHG) in the strained silicon waveguides. The second approach utilizes a silicon nitride ring resonator where the symmetry breaking occurs at the interface between silicon nitride core and silicon dioxide cladding. By satisfying the phase matching conditions and utilizing the cavity geometry, Levy *et al.* [30] demonstrate efficient SHG, generating a signal at 777 nm from a 1554 nm pump. In addition, $\chi^{(3)}$ -based third-harmonic generation is observed in the nitride ring resonators. These demonstrations offer additional paths toward further extending the operational wavelength range of these integrated devices.

3. Supercontinuum Generation

Over the past decade rapid progress has been made in supercontinuum generation (SCG) with the utilization of photonic crystal fibers [31]. A variety of applications exist for SCG, including spectroscopy, frequency metrology, optical coherence tomography, gas sensing, and pulse compression. Recently, the use of chip-scale devices has attracted interest, due to its potential for integration and high-volume, low-cost fabrication. Supercontinuum generation has been previously demonstrated in chalcogenide [32] and lithium niobate [33] platforms and both achieve broad spectral bandwidths. However, neither platform is readily compatible with existing CMOS processes. Alternatively, there has been work done on SCG in high-index silica glass waveguides 45-cm in length, with spectra spanning 300 nm and centered at 1550 nm [34].

Due to its high nonlinearity, silicon is an ideal platform for SCG. However, the effects of TPA limit the continuum bandwidth that can be achieved when pumped near the silicon bandgap [35], [36]. In 2011, Kuyken *et al.* [37] demonstrated SCG in silicon spanning three-quarters of an octave from 1535 to 2525 nm. This broad bandwidth is achieved by placing the pump at wavelengths near the half-bandgap energy of silicon, minimizing the effects of TPA and FCA. Alternatively, SCG has been demonstrated in silicon nitride waveguides. Although the nonlinearity of silicon nitride is smaller than that of silicon, it is an ideal platform for nonlinear optics at communication wavelengths due to the fact that it is still CMOS-compatible and does not suffer from the effects of TPA. Halir *et al.* [38] have shown SCG spanning 1.6 octaves from 665 to 2025 nm. These demonstrations offer promise towards the realization of a chip-scale supercontinuum source.

4. Parametric Oscillation and Frequency Comb Generation

In addition to linear waveguide structures, 2011 saw significant developments in integrated resonator structures for nonlinear optical processes. The high confinement of the optical field in the silicon waveguide structures combined with the enhancement due to the cavity geometry enables significant enhancements in the effective nonlinearity and results in highly efficient parametric processes with moderate pump powers. When the FWM gain exceeds the round-trip cavity loss, optical parametric oscillation occurs where the signal and idler pair at the cavity modes oscillates. At higher powers, cascaded parametric oscillation occurs enabling the generation of a broadband frequency comb. While frequency comb generation utilizing mode-locked femtosecond lasers is a mature technology [39], there has been significant interest over the past few years in developing a compact comb source based on parametric oscillation in high quality (Q) factor microresonators pumped by a single-frequency laser [40]. In particular, recent demonstrations using a CMOS-compatible, monolithically integrated resonator structures in high-index silica glass [41] and silicon nitride [42] offer potential as platforms for environmentally robust, ultracompact, stabilized frequency comb generation that can be used in applications such as spectroscopy, gas sensing, on-chip clock distribution for high-speed optical networks, remote clock synchronization, and photonic analog-to-digital conversion.

In 2011, there were significant advances in silicon-nitride-based frequency comb generation, both in generation and characterization [43], [44]. To enable efficient phase-matching for FWM over a broad bandwidth, the resonator cross section is engineered to create a broad spectral range with anomalous GVD, and researchers [44] demonstrated a parametric frequency comb spanning an octave of optical bandwidth from 1170 to 2350 nm (see Fig. 1). In parametric frequency comb

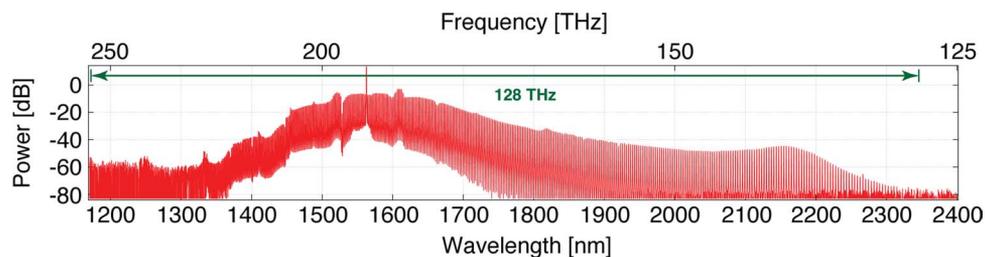


Fig. 1. Optical spectrum of an octave-spanning frequency comb in a silicon nitride resonator (from Okawachi *et al.* [44]).

generation, the wavelength of the single-frequency pump laser is red-detuned into a cavity resonance generating a signal-idler pair through FWM. As the pump wavelength is tuned deeper into resonance, the cavity power builds up further, resulting in cascaded FWM and higher-order degenerate and nondegenerate FWM processes increasing the frequency comb bandwidth and comb density. In addition, Okawachi *et al.* [44], characterize the RF amplitude noise of the comb and observe a 30 dB reduction in the noise as the comb is generated, indicating a transition to a phase-locked state. Foster *et al.* [43] characterized the comb quality and showed a comb equidistance of 3×10^{-15} relative to the optical frequency over a 115-nm bandwidth, which corresponds to a deviation 0.5 Hz over a 14.5-THz span. Since the dispersion of the resonator is highly dependent on the waveguide cross section, the dispersion and cavity free-spectral range are largely decoupled, allowing for independent control over the two parameters. This provides an advantage, as compared with whispering gallery resonators and allows for flexibility in choosing the pump wavelength, ranging from visible to midinfrared, and the comb spacing. Compared with femtosecond-laser-based frequency comb generation, parametric frequency comb generation is a technology that is not yet mature, and both theoretical and experimental studies are needed to understand the mechanisms governing the comb dynamics [45], [46]. Nevertheless, arbitrary waveform generation was achieved by Ferdous *et al.* [47] by controlling the spectral phase of each of the frequency comb lines involved in pulse shaping, and sub-ps pulses at a 230-GHz repetition rate were produced. As this technology matures over the next few years, we anticipate significant developments in on-chip frequency comb-based devices.

5. Conclusion

2011 saw significant advances in silicon-based devices utilizing the $\chi^{(2)}$ and $\chi^{(3)}$ nonlinearity at the traditional communication wavelengths and in the midinfrared. With the performance improvements shown in these devices, we anticipate the emergence of new applications utilizing chip-based nonlinear optics.

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